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PARTICLES MIGRATION IN LAMINAR BOUNDARY LAYER FLOW

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Abstract—The behavior of a suspension of neutrally buoyant solid particles in the moderate relative velocity region (close to the leading edge) in a laminar boundary layer along a flat plate, is investigated. The velocity fields of both the fluid and the solid particles and the density distribution of the solid particles are simultaneously obtained by the use of the Laser–Doppler technique.

It is shown that in this longitudinal region of the two-phase suspension boundary layer, the solid particles which initially lead the fluid decelerate as they move downstream, until at a certain vertical distance they lag the fluid. Clearly defined one-way transverse migration of particles directed away from the plate is characteristic of this region.

1. INTRODUCTION

The behavior of two-phase systems, specifically that of a liquid throughout which uniform size solid particles are distributed, covers a wide range of areas of great technical importance. Examples can be found in diverse areas, such as the separation of minerals in the field of mining engineering, flow through fluidized beds, environmental pollution. Lately, interest is developing in applying the fundamental theory of two-phase flow to hemodynamics and rheology of blood.

Conservation equations for two-phase flow were developed by various researchers. Due in part to the complexity of the equations, solutions of these equations were obtained mainly for dilute suspensions in flow systems such as boundary layer by Singleton (1965) or one-dimensional unsteady flows by Murray (1967). While all of these studies involved particles in fluid shear fields, the Stokes' drag was used as the sole interacting force between the two phases. The effect of a lift force acting on a particle due to its translation in a uniform shear field, which had been proposed by Saffman (1965), was later included in the work of Otterman & Lee (1969, 1970) for dilute suspensions of heavy particles. More recently, DiGiovanni (1971) and DiGiovanni & Lee (1973) extended the study of Otterman & Lee so as to include the effects of finite fractional particulate phase volume and the intrinsic material density of particles.

Most of the experimental studies reported in the literature were concerned with Poiseuille flow. Rigid spheres suspended in a steady laminer tube flow exhibit the tubular pinch effect; depending on the initial position of release, particles migrate radially inward or outward to an eccentric equilibrium radial position. It can be shown that single rigid spherical particles suspended in Stokesian flow have no tendency to move radially across stream lines, thus particle migratory motion must be inertial in origin. This migration phenomenon was later observed by other investigators and a thorough review was given by Brenner (1966).

Many conventional experimental techniques for the measurement of fluid velocity are not well suited to suspension flow in the boundary layer. To overcome the restrictions and to eliminate the errors inherent in employing conventional methods, Lee & Einav (1972) used the optical Laser–Doppler Anemometer (LDA) to measure the flow properties of a laminar suspension boundary layer in the far-downstream region. Since the LDA does not require the insertion of a mechanical probe into the flow field, it has many desirable features. The two-dimensional velocity fields of both the fluid and the solid particles and the density field of the solid particles were measured. In addition, it was observed that one-way migration of particles exists in the far downstream region of the boundary layer. This migration is directed towards its outer edge.

2. EXPERIMENTAL APPARATUS

Laser–Doppler anemometry

The theory and operating principles are covered by Durst & Whitelaw (1971), Foreman *et al.* (1966), Goldstein & Kreid (1967) and Yeh & Cummins (1964). Light from a laser which is scattered by small moving particles suspended in the fluid, undergoes a change in frequency. This Doppler shift is detected by mixing the scattered beam with a portion of the incident beam on the photocathode of a photomultiplier tube (PMT). The photocurrent is proportional to the Doppler signal and together with the known geometric arrangement of the system, velocity components of both the fluid and suspended particles may be determined.

The flow system

A schematic of the flow system is shown in figure 1. Water was pumped from the sump tank to the head tank. From the head tank it flowed through the water channel, which had an effective length of 6·1 m and inside dimensions 0·46 m wide and 0·30 m deep. The walls of the channel were made of optical grade plate glass.

A 2·44 m long flat plexiglass plate was horizontally mounted across a section of the channel where the flow was fully developed (figure 2(a)). The smooth plate was located at

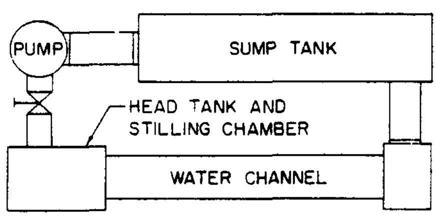

Figure 1. The water channel system (top view).

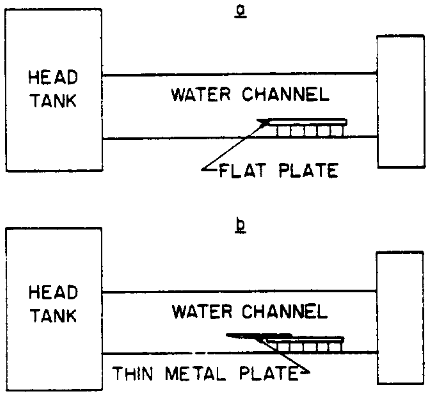

Figure 2(a, b). The flat plate.

an optimum vertical location as to minimize the undesirable effects of the bottom of the channel or the upper free water surface on the flow field.

The uniform-size glass spheres used in the experiment were of three sizes: 30, 50 and 100 μm. The densities of the particles and the water were matched by adding small quantities of salt to the water, in order to render the particles neutrally buoyant.

The tap water was filtered in order to limit the size of the natural contaminants in it to less than 2 μm for reasons which will be explained later.

The optical system

A detailed description of the optical system, as well as of the flow system, can be found in Lee & Einav (1972). A He–Ne Laser with minimum 15 mW output at 632.8 nm wavelength was used as the light source (figure 3). Its beam was split into two beams: the horizontal component and the vertical component. The two evenly divided beams were focused into a cylindrical volume approximately 100 μm in diameter and 250 μm in length.

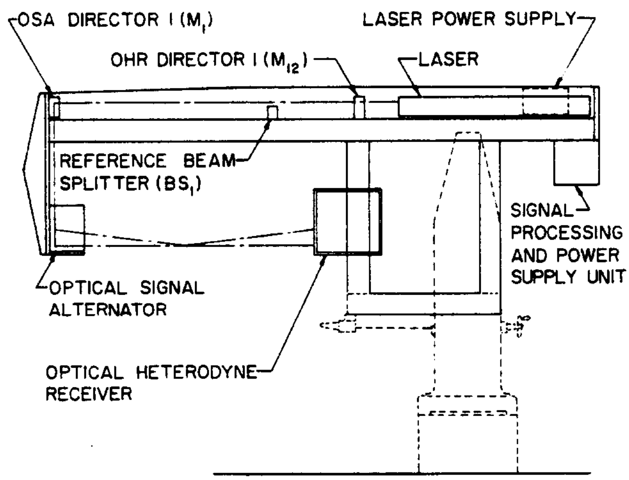

Figure 3. Laser–Doppler anemometer.

The exact size of this volume could be varied according to experimental requirements. The scattered beams were respectively mixed with an unaffected portion of the original laser beam in the optical heterodyne receiver (figure 3) and focused on the photomultiplier tube. The complete optical system was mounted on a precision traversing base, taken from a precision milling machine, which was capable of positioning the entire optical system relative to the water channel within 0.025 mm in all three directions.

3. INSTRUMENTATION

It was shown by Einav & Lee (1973) that two groups of scattering particles suspended in a fluid, which were different in size and moved with different velocities, produced separate Doppler signals in the frequency domain. These signals were distinctly different from each other in amplitude and in frequency. Thus, separation of the signals could be obtained by means of appropriate amplitude and frequency discrimination (Einav & Lee 1973). Since the tap water which was used as the carrying fluid was filtered, the size of the natural contaminants was less than 2 μm. The Doppler signal produced by these minute contaminants was reasoned to represent the flow field characteristics of the water, as it was demonstrated by Lee & Einav (1972) that there was practically no relative velocity between the particle contaminants and the water. The larger test particles suspended in the water produced signals different in amplitude and also in frequency when their velocity differed from that of the water.

The dual channel readout system shown in figure 4, was designed to obtain separately the characteristics of each of the two phases in the suspension flow; the horizontal velocity fields were read by one channel and the vertical by the other. The signal from each channel, after being amplified and filtered, was split into two branches: the fluid phase branch wherein the signal passed through a narrow-band filter to yield only the signal from the minute unfiltered contaminants in the water; and the particulate phase branch from which the high-amplitude signal from the larger test particles was amplified by the differential

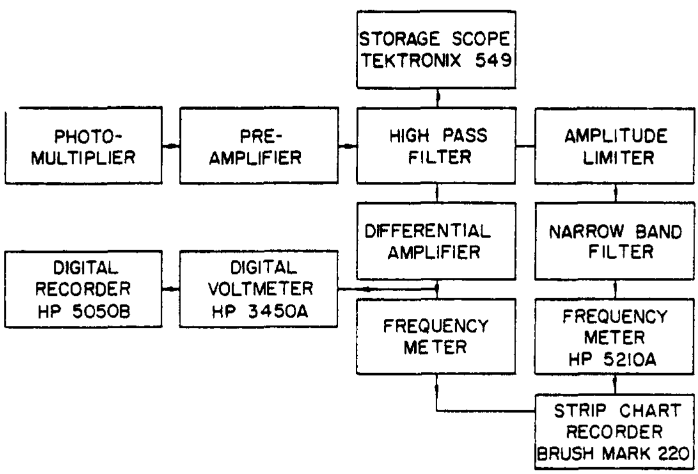

Figure 4. Readout system.

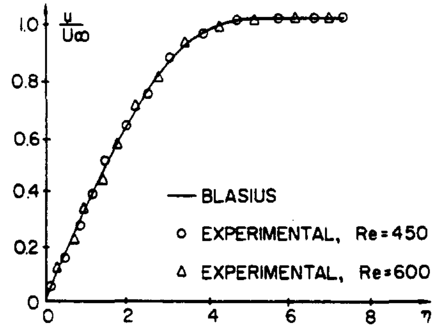

Figure 5. Single phase fluid horizontal velocity distribution.

amplifier. The frequencies of both signals were converted into voltage to be recorded on a strip chart recorder. The pulse height of the test particles signal was recorded on a digital recorder.

4. RESULTS AND DISCUSSION

Preliminary measurements were conducted in order to ascertain that the experimental apparatus operated satisfactorily. The boundary layer flow of water over a flat plate was investigated. Velocity of tap water containing only contaminant particles below 2 μm in size was measured and compared to the classical Blasius solution for incompressible laminar boundary layer flow along a flat plate (figures 5 and 6 where $\eta = y\sqrt{U_\infty/vx}$; $Re = U_\infty x/v$; x, y—coordinates parallel and transverse to the plate, respectively; u, v—velocity components in the x and y directions, respectively; v—kinematic viscosity of fluid and U_∞—free stream fluid velocity outside the boundary layer). The agreement, within 0.5 per cent in both longitudinal and transverse velocities, indicated the satisfactory operation of the system and confirmed that natural contaminants less than 2 μm in diameter served well as tracers for the fluid phase.

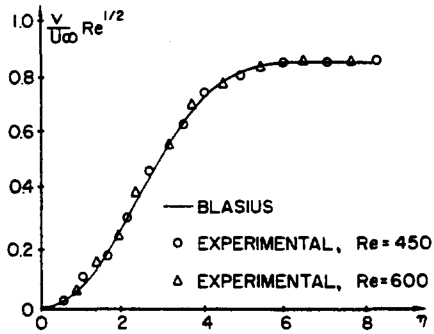

Figure 6. Single phase fluid vertical velocity distribution.

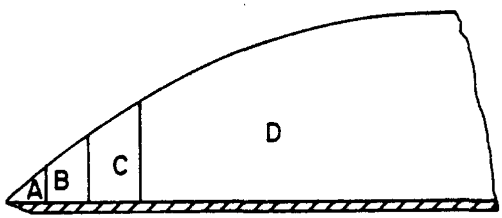

Figure 7. The longitudinal regions of the boundary layer.

The laminar boundary layer flow field can be divided into the following longitudinal regions (see figure 7):

A—The leading edge region;
B—The initial two-phase boundary layer flow (large relative velocity) region;
C—The moderate relative velocity region;
D—The far downstream (small relative velocity) region.

These regions are characterized by the non-dimensional longitudinal coordinate

$$\xi = x/\lambda.$$

x—Distance along the plate measured from the leading edge;
$\lambda = (2/9)(\rho_s a^2 U_\infty/\mu)$—Characteristic distance required for a particle to have its representative velocity relative to the surrounding fluid reduced to $1/e$ of its initial value (the relaxation length);
ρ_s—Intrinsic density of material which constitutes a discrete particle;
a—Particle radius;
U_∞—Free-stream fluid velocity outside boundary layer;
μ—Dynamic viscosity of the particle free fluid,

where

$\xi \simeq 0$—Defines region *A*;
$0 < \xi \ll 1$—Defines region *B*;
$\xi \simeq 1$—Defines region *C*;
$\xi \gg 1$—Defines region *D*.

This paper contains experimental results of laminar boundary layer flow in the moderate relative velocity region (Region *C*). The flow properties were investigated over the flat plate described above. To assure that the experimental data obtained was independent of the detailed physical geometry of the leading edge of the plate, a thin metal plate with a razor-like edge was attached to the plexiglass one (figure 2(b)) to form a new and sharper leading edge and the experiments repeated. Identical results were obtained for both cases.

The following flow properties were measured:

(a) Profiles of horizontal velocity for the fluid and the test particles, respectively;
(b) Profiles of vertical velocity for the fluid and the test particles, respectively;
(c) Profile of number density of test particles across the boundary layer.

These profiles were measured at several longitudinal locations for each of three different size test particles, each for two different free-stream particle concentrations, respectively. More than one free-stream velocity was utilized in each case.

Part of the experimental results of the velocity profiles and the test particle density distribution is given in figures 8–18 where m was the free-stream particle volumetric concentration. For the sake of comparison, the Blasius solution for a steady single-phase boundary layer flow was also shown. Presentation of the complete data was given by Einav (1972).

Effect of longitudinal location

Due to viscosity, the fluid in the boundary layer decelerated from its free-stream velocity to zero velocity at the wall. The solid particles, because of their inertia, could not instantaneously accommodate the deceleration of the fluid field and moved through the fluid with a relative velocity. In the region of moderate relative velocity, the particles moved faster than the fluid in that portion of the boundary layer closer to the wall, whereas the fluid led the particles in the outer portion, closer to the free-stream. The thickness of the layer in which the particles led the fluid decreased with the distance downstream of the plate (increasing ξ) until at some longitudinal location the particles then lagged the fluid

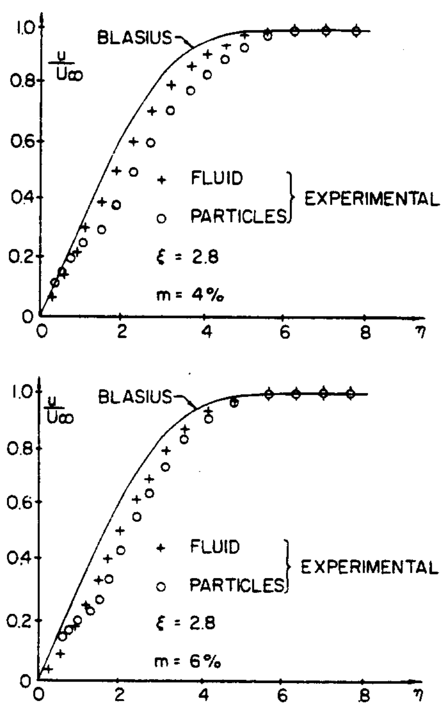

Figure 8. Horizontal velocity for 100 μm suspended particles. Particle concentrations $m = 4\%$ and $m = 6\%$; at a distance $\xi = 2.8$.

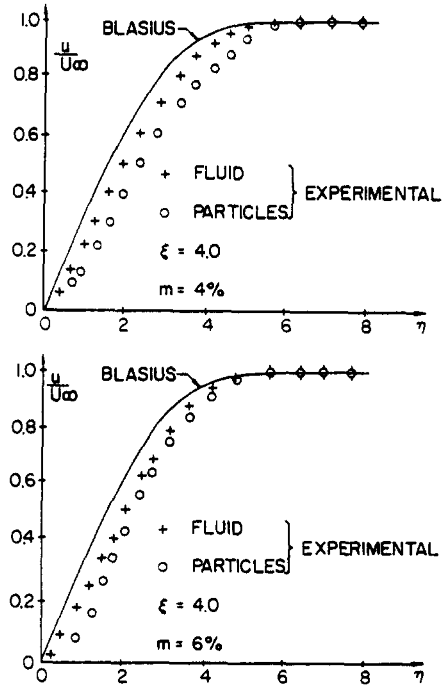

Figure 9. Horizontal velocity for 100 μm suspended particles. Particle concentrations $m = 4\%$ and $m = 6\%$, at a distance $\xi = 4.0$.

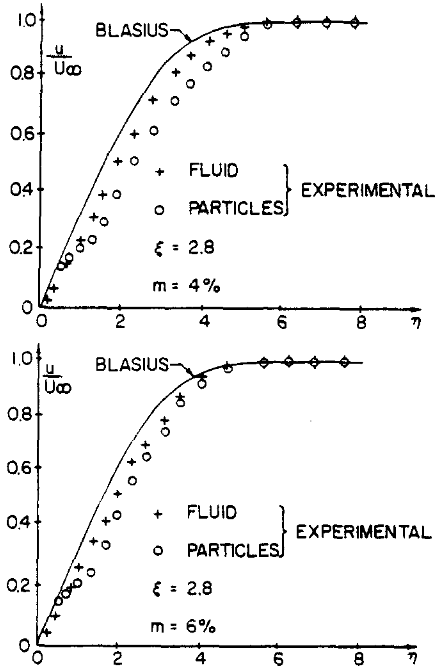

Figure 10. Horizontal velocity for 50 μm suspended particles. Particle concentrations $m = 4\%$ and $m = 6\%$; at a distance $\xi = 2.8$.

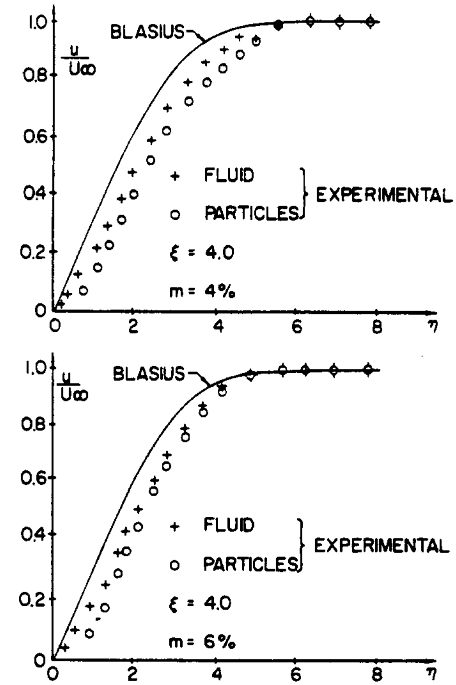

Figure 11. Horizontal velocity for 50 μm suspended particles. Particle concentrations $m = 4\%$ and $m = 6\%$; at a distance $\xi = 4.0$.

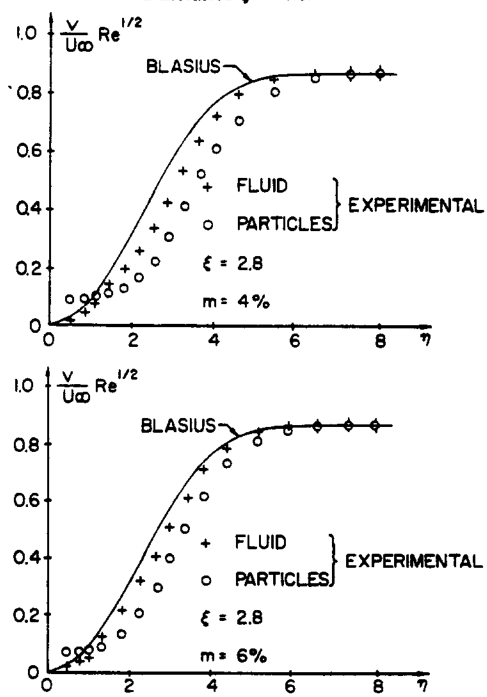

Figure 12. Vertical velocity for 100 μm suspended particles. Particle concentrations $m = 4\%$ and $m = 6\%$; at a distance $\xi = 2.8$.

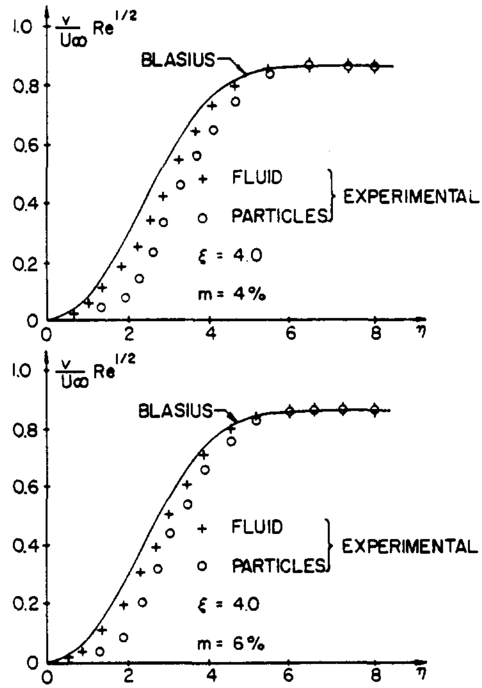

Figure 13. Vertical velocity for 100 μm suspended particles. Particle concentration $m = 4\%$ and $m = 6\%$; at a distance $\xi = 4.0$.

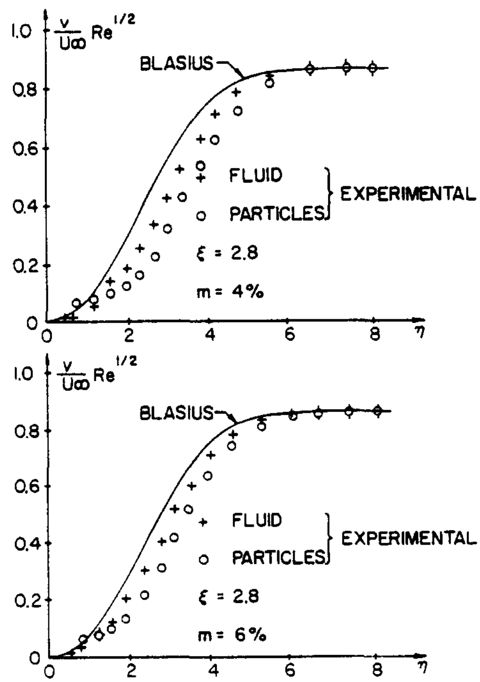

Figure 14. Vertical velocity for 50 μm suspended particles. Particle concentrations $m = 4\%$ and $m = 6\%$; at a distance $\xi = 2.8$.

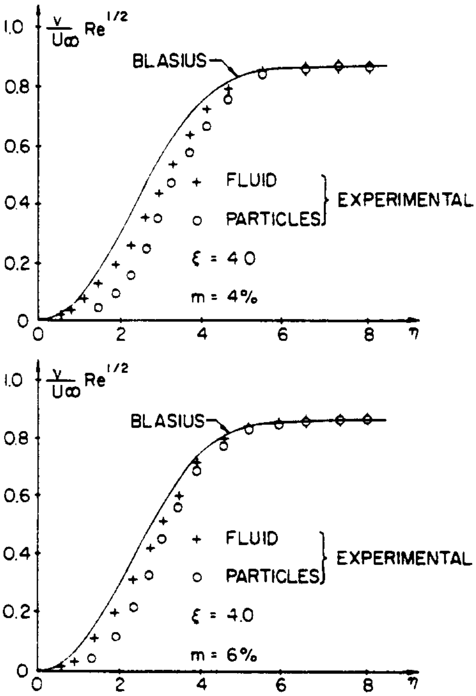

Figure 15. Vertical velocity for 50 μm suspended particles. Particle concentrations $m = 4\%$ and $m = 6\%$; at a distance $\xi = 4.0$.

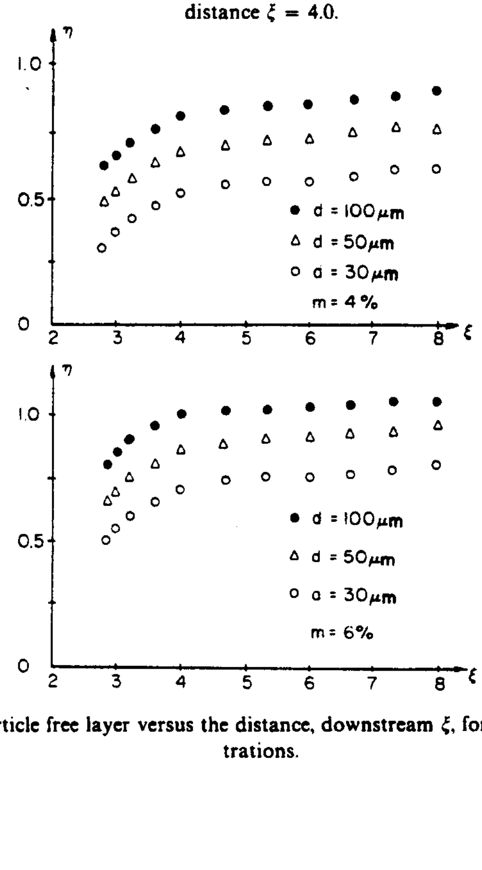

Figure 16. Thickness of particle free layer versus the distance, downstream ξ, for various diameters and concentrations.

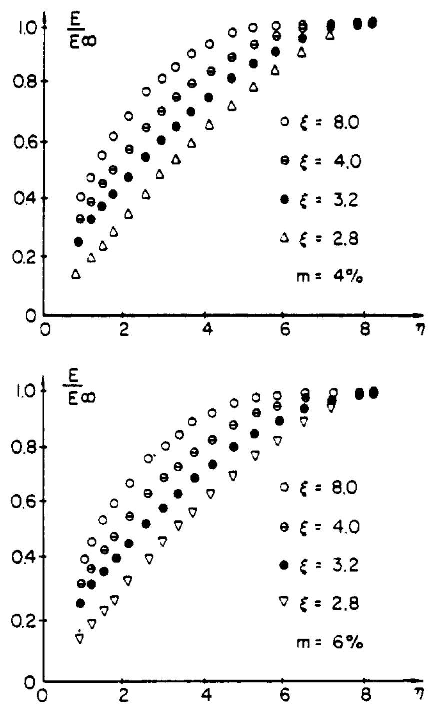

Figure 17. Density distribution for 100 μm suspended particles.

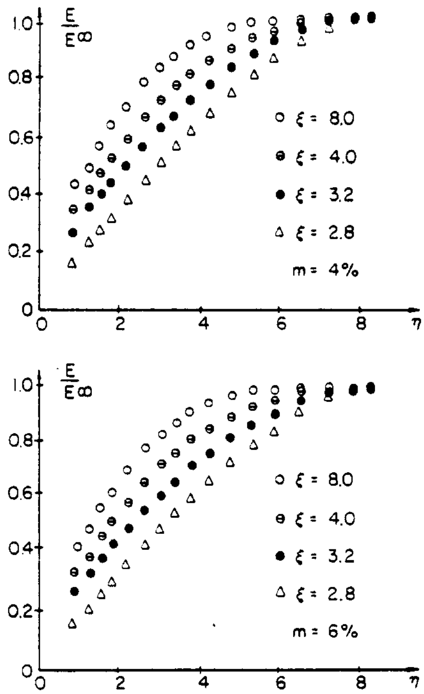

Figure 18. Density distribution for 50 μm suspended particles.

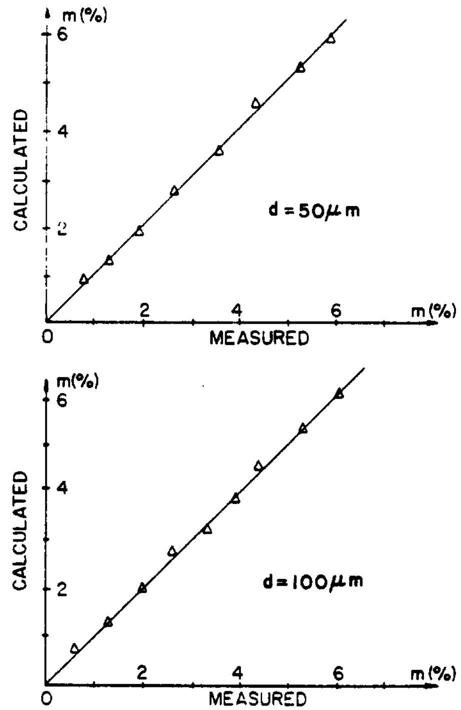

Figure 19. Particle concentration, measured vs. calculated.

the remainder of the boundary layer. Moreover, the solid particles which moved with the fluid in the free-stream were found to move away from the plate more rapidly than the fluid upon entering the moderate relative velocity region of the boundary layer. As one moved along the plate the particles decelerated in the boundary layer and their vertical velocity eventually became smaller than that of the fluid. The longitudinal location after which the particles lagged the fluid horizontally did not necessarily coincide with the location at which the particles lagged the fluid vertically.

The thickness of the particle-free layer adjacent to the wall measured from the vertical velocity component seemed in several cases to be slightly larger than the one measured from the horizontal velocity component. This was primarily due to the extremely low vertical velocities encountered in the inner portion of this layer which yielded too low a Doppler shifted frequency to be detected with the electronics used. For the fluid phase, on the other hand, it was possible to obtain a reading down to a distance as small as 35 μm from the wall, a limit set by the smallest size of the optical measuring volume and the noise level of the readout system.

Effect of free-stream particle concentration

It was observed that the relative velocity between the phases decreased as the particle concentration increased. The region in which the particles lagged the fluid throughout the boundary layer moved to the leading edge as the particle concentration increased.

It was also observed that the thickness of the particle free layer increased with the free-stream particle concentration. This could be attributed to the increase of the vertical particle velocity away from the wall with increasing particle concentration.

The steady sharp signals obtained throughout the range of free-stream particle concentration studied indicated the absence of significant particle–particle collisions.

Effect of the size of particles

Three different sizes of test particles were used in the present experiments: 30, 50 and 100 μm in diameter. Each experiment was conducted with particles of one of these sizes. The relative velocity and thickness of the particle free layer adjacent to the wall were found to increase with an increase in the size of the test particles (see figure 16 where d is the particle diameter).

Particle density distribution

The test particle density distribution E/E_∞ (or concentration distribution) for the moderate slip region was given in figures 17–18 for various longitudinal stations, free-stream test particle concentrations and sizes of test particles. It was known that the intensity of the scattered beam from particles of uniform size varied directly with the concentration of the particles. However, the data obtained from the amplitude of the output intensity signal gave only a measure of the relative particle concentration. Thus, a new supplementary experimental scheme was introduced to independently measure the particle concentration. The size of the scattering volume was reduced so as to allow scattering from only one test particle of a known size at a time. Each particle passing through the scattering volume produced a Doppler signal pulse that could be recorded on the strip chart recorder. The time interval between successive pulses and the frequency of the pulse (indicative of the velocity of the particles) gave the spacing between two successive particles. With known particle size the particle concentration could be calculated. This technique was preferable to the alternate technique of light attenuation, as the latter gives an integrated reading whereas the former gives the local particle concentration.

This particle counting technique was used to measure the free-stream particle concentration. The results were compared to the free-stream particle concentration obtained independently from reservoir calibration, as shown in figure 19. The close agreement indicated the reliability of this measuring technique.

It was noted from the plots of particle concentration distribution that one-way migration existed in the moderate slip region of the boundary layer and was directed towards its outer edge.

5. CONCLUSIONS

At present, there exists no theoretical analysis which describes the moderate slip region of the laminar two-phase suspension boundary layer flow. The ranges of values of the significant dimensionless parameters covered in the present experiments are:

(a) Main flow Reynolds number—$(Re)_\Delta = 250 \div 450$

$$(Re)_\Delta = U_\infty \Delta / v$$

Δ—Characteristic boundary layer thickness;

(b) Particle Reynolds number based on the relative velocity—$(Re)_p = 0.12 \div 3{\cdot}8$

$$(Re)_p = U_\infty (2a)/v;$$

(c) Shear Reynolds number—$(Re)_S = 95 \div 325$

$$(Re)_S = S(2a)^2/v$$

S—Characteristic shear rate in the boundary layer.

The results of the experiments showed that in the moderate longitudinal relative velocity region of the two-phase suspension boundary layer, the particles which initially led the fluid decelerated as they moved downstream. As the vertical distance from the plate increased, the horizontal relative velocity changed signs.

This increase in the vertical relative velocity was felt to be produced by the lateral force on a particle experiencing horizontal relative velocity in a fluid shear field. Clearly defined one-way transverse migration of particles directed away from the plate was characteristic of this longitudinal region of the boundary layer.

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Sommaire—On étudie le comportement d'une suspension de particules solides neutralement flottantes dans la région de vélocité relative modérée (proche du rebord de front) dans une couche limitrophe laminaire au long d'une plaque plate. Les champs de vélocité du fluide et des particules solides et la répartition de densité des particules solides sont obtenus simultanément par l'emploi de la technique Laser-Doppler.

Il est montré que dans cette région longitudinale de la couche limitrophe de suspension à deux phases, les particules solides qui mènent initiallement le fluide décélèrent à mesure qu'elles se meuvent en aval jusqu'au moment où à une certaine distance verticale elles sont en retard par rapport au fluide. Une migration transversale uni-directionelle clairement définie des particules au-delà de la plaque est caractéristique de cette région.

Auszug—Das Verhalten einer Suspension von festen, neutral schwimmenden Teilchen in der langsamen, relativen Geschwindigkeitsgegend (nahe der fürenden Kante) wird in einer laminaren Grenzschicht entlang einer ebenen Platte untersucht. Die Geschwindigkeitsfelder sowohl der flüssigen wie der festen Teilchen und die Dichteverteilung der festen Teilchen werden gleichzeitig mit dem Laser-Dopplerverfahren erhalten.

Es wird gezeigt, daß in dieser längslaufenden Region der zweiphasigen Suspensionsgrenzschicht die festen Teilchen, welche anfänglich der Flüssigkeit vorangehen, sich verlangsamen, während sie sich flußabwärts bewegen, bis sie bei einem bestimmten vertikalen Abstand der Flüssigkeit nacheilen. Klar definierte Einbahnquerwanderung von Teilchen, von der Platte fortgerichtet, ist für diese Region charakteristisch.

Резюме—Исследуется поведение взвеси нейтрально плавучих твердых частиц в зоне сравнительной скорости ламинарного граничного слоя (вблизи передней кромки) по плоскому листу. Лэзер-Допплеровским методом получили поле скоростей как для твердых так и для жидких частиц и также распределение по плотности твердых частиц.

Нашли, что в продольной зоне двухфазного граничного слоя взвеси твердые частицы, которые первоначально вели жидкие, по мере передвижения по направлению потока замедляют свое движение до тех пор, пока на определенном расстоянии они не начнут отставать от жидких частиц. В этой зоне ясно заметно поперечное перемещение частиц по направлению от листа.